Bulk Solid Handling

Rodrigo Gutierrez
INTRODUCCION
Throughout the world, the handling and processing of powders and bulk materials are key operations in a great number and variety of industries. Such industries include those associated with MINING, MINERAL PROCESSING, chemical processing, agriculture, power generation, food processing, manufacturing and pharmaceutical production.

The foundations of process and handling plant design lies in the determination of the bulk solid flow properties and the correct interpretation of these properties in relation to the particular applications. The design of bulk solids handling plant requires a knowledge of the strength and flow properties of the bulk solids under operating conditions.
The latter conditions include loading and consolidation for instantaneous and extended time storage as well as environmental factors such as temperature, moisture and humidity.

There are now well established laboratory test procedures for determining the necessary flow properties.
DETERMINATION OF BULK SOLIDS FLOW PROPERTIES

Focusing specifically on bin, hopper and stockpile design, the laboratory tests aim to duplicate field conditions and provide the designer with such parameters as:

(i) Yield loci and flow functions $FF$ for instantaneous and time storage conditions for the range of moisture contents and, as relevant, temperatures occurring in practice. The flow functions represent the variation of unconfined yield strength with major consolidation stress as occurs during storage and flow.

(ii) Effective angle of internal friction $\delta, \varphi_e$ as a function of major consolidation stress
(iii) Static angle of internal friction $\phi_t$ as a function of major consolidation stress.

(iv) Wall friction angles $\phi$ as a function of normal stress for different bin and chute wall materials and finishes.

(v) Bulk density $\rho$ as a function of major consolidation stress.

(vi) Solids density.

(vii) Permeability of the solids as a function of major consolidation stress.
Jenike Type Direct Shear Tester

(a) Pre-Consolidation of Sample

(b) Shear Consolidation followed by Shear to Failure
B3: incipient flow

C: incipient flow

A: consolidation
- $ff_c < 1$ not flowing
- $1 < ff_c < 2$ very cohesive
- $2 < ff_c < 4$ cohesive
- $4 < ff_c < 10$ easy-flowing
- $10 < ff_c$ free flowing

$ff_c = \frac{\sigma_1}{\sigma_c}$
THEORY AND BACKGROUND

- Follows a solid element as it passes through a vertical flow channel.
- As the materials flows, ratio of major stresses for a bulk solids of constant moisture content only vary a little.
- Based on Mohr’s stress circle theory.

**Major Properties of Bulk Solids:**

- Can transfer shearing stresses under static conditions (static angle of friction greater than zero).
- Many solids, when consolidated, possess cohesive strength and retain their shape.
- Shearing stresses in a ‘flowing’ bulk solid are independent of the rate of shear and dependent on the mean (consolidation) pressure.
Fig. 3.11. Course of shear stress, \( \tau \), when shearing differently preconsolidated samples at identical normal stress, \( \sigma \); a. underconsolidated specimen, b. overconsolidated specimen.
Fig. 3.13. Preshear point and shear point in a $\sigma, \tau$ diagram: a. cohesive bulk solid; b. cohesionless, free-flowing bulk solid

Fig. 3.14. Determination of the yield limit from the measured shear stresses
Fig. 3.16. Yield locus and Mohr stress circles defining the unconfined yield strength, $\sigma_c$, and the consolidation stress, $\sigma_2$; analogy to the uniaxial compression test.
Fig. 3.20. Yield locus and flow properties
SHEAR TEST RESULTS (Preshear and Failure-Shear)

3 Steps:
1. Pre-Consolidation to create a uniform sample
2. Pre-Shear (Consolidation) to establish shear plane under steady state
3. Failure shear to establish failure plane and yield locus

The material has climbed steeply, reached a plateau before shearing completely and falling away. OVER-CONSOLIDATED

The material has climbed slowly without reaching a steady shear. The sample will fall away eventually due to the loss of area when the cell cross on other side. UNDER-CONSOLIDATED

The material has climbed not quite as steeply as the Over-Consolidated test. It has reached a plateau and has shown a steady shear about where the rings become concentric. ACCEPTABLE
INSTANTANEOUS YIELD LOCUS

Mohr Circle:
Major Circle is tangential to YL and goes through Consolidation Point.
Failure Circle goes through origin and is tangential to YL.

Yield Loci:
Normal Stress (x) determined by the failure shear load (less than pre-shear load).
Shear Stress (y) is actual failure shear reading value.

Consolidation Point:
Normal Stress (x) determined by pre-shear load plus weight carrier and WASP.
Shear Stress (y) derived from average of all (usually 4-6) pre-shear steady state stress.

Instantaneous Yield Locus
Calculated YL (solid) - Defined YL (dotted)
EXAMPLE: 3 POINTS FLOW FUNCTION

Defined yield loci, straight line through consolidation points and flow function.

- Procedure from previous slide yields a single point on the so-called "flow function".
- Shear Stress is derived from $\sigma_2$ of the relevant failure Mohr circle.
- Normal Stress is derived from $\sigma_1$ of the relevant large Mohr circle.

Multiple Points on the flow function are obtained by varying the amount of pre-shear leading to the consolidation point.
INTERNAL FRICTION ANGLES

Effective Angle of Internal Friction ($\delta$):
The slope angle of the Effective Yield Locus (EYL) which is a line from the origin, tangential to the Major Mohr circle.

Kinematic Angle of Internal Friction ($\phi$):
The slope angle of the Instantaneous Yield Locus (IYL) at the point of intersection with the Mohr circle through the origin.
At a boundary or surface, such as that of a hopper or chute, the flow is characterised by the Wall Yield Locus (WYL) which relates the shear stress at the wall to the corresponding normal stress, or pressure. The WYL is usually obtained using a Jenike direct shear apparatus.
Wall Yield Locus and Wall Friction Angle

- Wall Yield Locus (WYL)
- Wall Friction Angle
- Cohesion $\tau_0$
- Adhesion $\sigma_0$
- Tension
- Compression
CHARACTERISATION FOR HOPPER AND STOCKPILE DESIGN

The procedures for the design of handling plant, such as storage bins, gravity reclaim stockpiles, feeders and chutes are well established and follow the four basic steps:

(i) Determination of the strength and flow properties of the bulk solids for the worst likely flow conditions expected to occur in practice.

(ii) Determination of the bin, stockpile, feeder or chute geometry to give the desired capacity, to provide a flow pattern with acceptable characteristics and to ensure that discharge is reliable and predictable.

(iii) Estimation of the loadings on the bin and hopper walls and on the feeders and chutes under operating conditions.

(iv) Design and detailing of the handling plant including the structure and equipment.
MODES OF FLOW IN BINS

(a) Mass-Flow
First in, First Out

(b) Funnel-Flow
First in, Last Out
COMMON BIN GEOMETRIES

Figure 3. Variations of Mass-Flow Bins

Figure 4. Typical Funnel-Flow Bin

Figure 5. Expanded Flow
Jenike postulated that the major principal stress \( \sigma_1 \) in the lower hopper part is proportional to the distance, \( r \), from the virtual hopper apex ("radial stress field").

\[
\sigma_1 = r \cdot g \cdot \rho_b \cdot s(\Theta', \Theta, \varphi_x, \varphi_e) \cdot (1 + \sin \varphi_e)
\]

Principal Stress at any position in the hopper

A solution of the system of differential equations exists only for specific combinations of parameters, \( \Theta \), \( \varphi_e \), and \( \varphi_x \). Thus, only for these conditions mass flow will occur. If the hopper wall is not steep enough, no solution exists fulfilling the condition of bulk solid moving along the hopper wall (mobilization of wall friction), and, thus, a stagnant zone will form (funnel flow).

\( \varphi_x \): angle of wall friction,
In the case of conical hoppers, it is recommended that, as a safety precaution, the hopper half angle be, generally, 30° less than the limiting value.

Consider, for example, a conical hopper handling coal with $\delta=50^\circ$. If the hopper is of mild steel with a mill scale surface and some possible corrosion, flow property tests indicate that the angle $\phi$ is likely to be approximately $30^\circ$. On this basis, $\alpha=14^\circ-19^\circ=11^\circ$, which makes for a very steep hopper. If the hopper is lined with stainless steel type 304 with 2B finish, flow property tests indicate that the friction angle $\phi$ is likely to be $20^\circ$ or even lower. On this basis $\alpha=25^\circ-3^\circ=22^\circ$. The corresponding angles for plane-flow are $\alpha=22^\circ$ for $\phi=30^\circ$ and $\alpha=34^\circ$ for $\phi=70^\circ$. This shows the advantage of plane-flow over axi-symmetric or conical flow since there is a substantial increase in the hopper half-angle. However, as previously discussed, the advantage is offset, to some extent, by the fact that plane-flow or wedge-shaped hoppers have long, slotted openings, which makes it difficult to obtain uniform drawdown when feeding along the slot.
MASS FLOW DESIGN DETERMINATION OF MINIMUM OUTLET

Fig. 10.8. Model for determination of minimum outlet size, \( d_{crit} \), to avoid arching

A stable arch is only possible in that part of the hopper where the unconfined yield strength is greater than the stress that would exist in a stable arch \((\sigma_c > \sigma_1')\), i.e., beneath the point of intersection of the \(\sigma_c\) curve with the \(\sigma_1'\) line (Fig. 10.8). Above the point of intersection the unconfined yield strength is smaller than the major stress in the arch, i.e., the material will flow.

\[
\sigma_1' = \frac{2r \cdot \sin \Theta \cdot g \cdot \rho_b}{1 + m}
\]
- Aim of Mass Flow Design is to determine the minimum cohesive arching dimension $B$
- Opening dimension for mass flow must be large enough to prevent an arch from forming
- A cohesive arch forms as a result of high bulk strength due to consolidation
- A mechanical arch occurs as a result of particle interlocking
- In order to prevent mechanical arches from forming, $B$ should be at least 3 (plane) or 5 (conical) times the maximum lump size
FLOW FACTOR

- For mass flow, it is agreed that a solid will flow from a hopper if no arch develops across this channel.
- In funnel flow bin storage, it is also necessary to assure that the solid is unable to sustain an empty vertical pipe (rathole).
- Flow Function FF is a bulk solid parameter.
- The arch stress $\bar{\sigma}_1$ is related to the major consolidation stress by the Flow Factor $f_f = \frac{\sigma_1}{\sigma_1'}$.

$$f_f = \frac{\bar{\sigma}_1}{\sigma_1'} = \text{const} = (1 + m) \cdot s(\Theta, \varphi_x, \varphi_e) \cdot \frac{1 + \sin \varphi_e}{2 \sin \Theta}$$

- Flow – No Flow Scenario: Gravity flow of a solid in a channel will occur provided the unconfined yield strength $\sigma_c$ under given consolidation pressure is insufficient to produce a flow obstruction (arch).
Critical Arching Dimensions can be calculated as follows:

\[ B_{cr} = \frac{\sigma_1 H(\alpha)}{\rho g} = \frac{\sigma_1 H(\alpha)}{f_f \rho g} \]

Empirical Function \( H(\alpha) \):

- \( m = 1 \) Axi-Symmetric Flow
- \( m = 0 \) Planar Flow

Graph showing the relationship between \( H(\alpha) \) and \( \alpha \):

- Square Outlet
- Planar Flow

Graph illustrating the relationship between \( \sigma_1 \) and \( \sigma_c \):

- No Flow
- Flow

\[ f_f = \frac{\sigma_1}{\sigma_i} \]

Symbols:
- \( \alpha \)
- \( B_{cr} \)
- \( \Delta V \)
- \( \rho \)
- \( g \)
Fig. 10.12. Flow function and time flow functions: major stress in a stable arch

\[ \sigma_i' = \frac{\sigma_i}{\text{ff}} \]

Figure 31. Flow Factor Chart for Conical Hopper - $\delta = 50^\circ$
MASS-FLOW DESIGN EXAMPLE

In order to illustrate the procedures for mass-flow hopper design, the case of a ‘Run-of-Mine’ (ROM) coal at 13% moisture content, wet basis, is considered. The Flow properties are given in Figures 36 and 37. The lower graphs of Figure 36 shows the instantaneous and three day time Flow Functions. The middle graphs show the static $\phi$, and effective $\delta$ angles of internal friction. The upper graph shows the bulk density. In all cases the independent variable is the major consolidation stress $\sigma_1$. Figure 37 presents the wall friction data for the ROM coal in contact with mild steel with mill scale finish and stainless steel Type 304 with 2B finish. The upper graphs depict the Wall Yield Loci (WYL) while the lower graphs show the Wall Friction Angles ($\phi$), both graphs being plotted against the normal pressure at the boundary surface. It is proposed to determine the required geometry of a conical or axi-symmetric mass-flow hopper. The procedure is as follows:

Limiting or Critical Geometry

(i) Estimate $\delta$ and $\phi$ at the outlet:
Assume $\delta = 60^\circ$, $\phi = 23^\circ$

From Figure 33, $\alpha = 20^\circ$ and $ff = 1.21$

(ii) Draw $ff = 1.21$ line on the FF graph of Figure 36. The intersection point is $\sigma_1 = 4.5$ kPa and $\bar{\sigma}_1 = 3.5$ kPa
Checking $\delta=60^0$ and, based on Figure 12(c), $\phi=23^0$. Hence it is not necessary to perform any iterations.

(iii) Determine Minimum Opening Dimension $B$

From Figure 36, $\rho = 0.95$ t/m$^3$

From Figure 30, $H(\alpha) = 2.32$

Substitution into equation (12) gives $B = 0.87$ m
Figure 36. Flow Properties for ROM Coal at 13% Moisture Content
Figure 10.14. Determination of the wall normal stress in a mass flow hopper: a. wall normal stress, \( \sigma_n \), and major principal stress, \( \sigma_1 \), at hopper wall; b. Mohr stress circle representing the stresses at the wall.
**Draw-Down in Gravity Reclalm Stockpile**

For complete draw-down, feeder size would need to be too large.

Better solution is to use 2 optimally spaced reclaim hoppers and feeders to maximize draw-down and live capacity.
FUNNEL FLOW DESIGN

Lower Bound

\[ \sigma_{l''} = \frac{D \cdot g \cdot \rho_b}{f(\varphi_i)} \]

\[ \sigma_{l} = \frac{1 + \sin \varphi_e}{4 \cdot \sin \varphi_e} \cdot D \cdot g \cdot \rho_b \]

\[ f_{fp} = \frac{\sigma_l}{\sigma_{l''}} = \frac{1 + \sin \varphi_e \cdot f(\varphi_i)}{4 \cdot \sin \varphi_e} \]

If the calculation leads to a value of $f_{fp} < 1.7$, a fixed value of $f_{fp} = 1.7$ has to be used.

Fig. 10.15. Stable rathole and circumferential stress, $\sigma_{l''}$
\[ D_{\text{crit}} = f(\phi_i) \frac{\sigma_{c,\text{crit}}}{g \cdot \rho_{b,\text{crit}}} \]
Upper Bound

\[ \sigma_{1,\text{crit}} = \sigma_v = \frac{\rho_b g A}{K \tan \phi_x U} \cdot \left[ 1 - e^{\frac{-K \tan \phi_x U h_f}{A}} \right] \]

Fig. 10.18. Distribution of vertical stress, \( \sigma_v \), in a funnel flow silo for filling conditions (stresses in the hopper approximated with Janssen’s equation, see Sect. 9.2.1)
The computation of draw-down hD and live capacity involves the following steps:

(a) Determination of the relationship between the rathole diameter Df versus effective head of solids hf. This determination is derived entirely from the flow properties of the bulk solid without reference to a particular bin or stockpile geometry. Usually Df versus hf is expressed in graphical form.

(b) Using the information in (a), the relationship between the rathole dimension Df and draw-down hD can be obtained for a particular bin or stockpile geometry. Again, graphical representation is usually used. With this information, the dimensions of the outlet, or, in the case of expanded flow, the dimensions of the lower mass-flow hopper at the transition with the funnel-flow section, can be selected.

(c) For a particular geometry for the bin outlet or mass-flow hopper transition, the shape of the rathole is estimated and this information is used in conjunction with the Df versus hf relationship of (b) to determine the actual draw-down (d). The live capacity is then estimated by calculating the volume of the craters formed after draw-down has occurred.
Rathole Geometry
(a) Scale Model Stockpile

(b) Computer Simulation – Hoop Stress Theory
Funnel Flow Design – (Jenike)

\[ D_f = \frac{\sigma_c G(\phi)}{\gamma} \]
Calculation of Draw-Down

\[ hf = \frac{R}{K_j \tan \phi} \left( 1 - e^{-\frac{K_j \tan \phi}{z/R}} \right) \quad (3) \]

where: \( R = \frac{\text{area of bin cross-section}}{\text{perimeter of bin cross-section}} \)

\( K_j = \text{ratio of horizontal to vertical pressure in the bin.} \)

Assumed to be 0.4.

\( \phi = \text{wall friction angle in degrees} \)

\( z = \text{actual head of solids} \)

For a circular bin \( R = \frac{D}{4} \quad (4) \)

Referring to Figure 1 and substituting \( h_D \) for \( z \) in equation (3) and solving for \( h_D \) gives

\[ h_D = \frac{D}{4 K_j \tan \phi} \ln \left[ \frac{D}{D - 4 K_j hf \tan \phi} \right] \quad (5) \]
Upper Bound Value of Critical Rathole Dimension

The upper bound rathole dimensions are determined on the basis of the time flow function \( FF_t \) and the major consolidation stress \( \sigma_1 \).

\[
D_f = \frac{\sigma_c G(\phi_t)}{\gamma}
\]
**Funnel Flow Design**

A funnel flow silo with conical hopper has to be designed whereby both the lower and upper bounds of the rathole diameter have to be determined. The bulk solid is assumed to exhibit no time consolidation. The diameter of the silo is $D = 3$ m, the maximum filling level for filling conditions, i.e., before material is discharged, is $hf = 6$ m.

<table>
<thead>
<tr>
<th>Yield locus no.</th>
<th>$\sigma_s$ [Pa]</th>
<th>$\sigma_c$ [Pa]</th>
<th>$\rho_s$ [kg/m$^3$]</th>
<th>$\phi_c$ [$^\circ$]</th>
<th>$\phi_{lim}$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4200</td>
<td>3100</td>
<td>880</td>
<td>50</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>8700</td>
<td>4350</td>
<td>935</td>
<td>50</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>14000</td>
<td>5760</td>
<td>965</td>
<td>50</td>
<td>39</td>
</tr>
<tr>
<td>4</td>
<td>27500</td>
<td>7050</td>
<td>980</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>
Lower Bound

Assuming $\phi_{lin} = 30^\circ$, $f(\phi_i = \phi_{lin} = 30^\circ) = 2.4$

Flow factor $ff_p = 1.38$. Since $ff_p$ should not be smaller than 1.7, it follows $ff_p = 1.7$.

<table>
<thead>
<tr>
<th>$\sigma_{1,\text{crit}}$ [Pa]</th>
<th>$\sigma_{e,\text{crit}}$ [Pa]</th>
<th>$\rho_{b,\text{crit}}$ [kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6200</td>
<td>3650</td>
<td>910</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi_{e,\text{crit}}$ [$^\circ$]</th>
<th>$\phi_{lin,\text{crit}}$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>30</td>
</tr>
</tbody>
</table>

$D_{\text{crit}} = f(\phi_i) \frac{\sigma_{e,\text{crit}}}{g \cdot \rho_{b,\text{crit}}} = 0.98$ m.
Upper Bound

\[ \sigma_{1, crit} = \sigma_v = \frac{\rho_b g A}{K \tan \phi_x U} \left[ 1 - e^{-\frac{K \tan \phi_x U h_f}{A}} \right] \]

Filling height, \( h_f = 6 \) m;

Ratio of cross-section to perimeter, \( A/U = D/4 = 0.75 \) m.

Wall friction angle, \( \phi_x = 25^\circ \),

maximum bulk density, \( \rho_b = 980 \) kg/m\(^3\),

\( K = 0.5 \)

Stress \( \sigma_{1, crit} = 26.14 \) kPa.

<table>
<thead>
<tr>
<th>( \sigma_{1, crit} [\text{Pa}] )</th>
<th>( \sigma_c, crit [\text{Pa}] )</th>
<th>( \rho_b, crit [\text{kg/m}^3] )</th>
<th>( \phi_{lin, crit} [^\circ] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>26140</td>
<td>6900</td>
<td>979</td>
<td>40</td>
</tr>
</tbody>
</table>

\[ D_{crit} = f(\phi_i) \frac{\sigma_{c, crit}}{g \cdot \rho_{b, crit}} = 2.59 \text{ m.} \]
BIN WALL LOADS

Pressures Acting in Mass-Flow Bins
Stresses in vertical channels (Janssen’s approach)

\[ \tan \varphi_x = \frac{\tau_w}{\sigma_h}, \]
\[ K = \frac{\sigma_h}{\sigma_v}, \]

Fig. 9.8. Slice element (bulk solid) in the vertical section

\[ A\sigma_v + g \rho_b A \, dz = A(\sigma_v + d\sigma_v) + \tau_w U \, dz \]
\[ p_v = \frac{\gamma r_c}{\mu k} \left[ 1 - e^{-(\mu k y/r_c)} \right] + p_{vo} e^{-(\mu k y/r_c)} \]  

\( r_c \) = characteristic radius of container  
\( \gamma \) = \( \rho g \) = Bulk specific weight  
\( \rho \) = bulk material density  
\( g \) = acceleration due to gravity  
\( \mu \) = \( \tan \phi_w \) = coefficient of wall friction  
\( \phi_w \) = wall friction angle  
\( p_{vo} \) = average surcharge pressure due to natural surcharge of material caused by filling.  
\( k \) = \( \frac{p_{ni}}{p_v} \) = lateral pressure ratio
Characteristic Radius $r_c$

For Round or Square Container

\[ r_c = \frac{D}{4} \text{ for round or square container} \]

\[ D = \text{Bin diameter or Width} \]

For Rectangular Container of length $L$ and width $D$

<table>
<thead>
<tr>
<th>( \frac{L}{D} )</th>
<th>For Short Side</th>
<th>For Long Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.27D</td>
<td>0.30D</td>
</tr>
<tr>
<td>2.0</td>
<td>0.30D</td>
<td>0.33D</td>
</tr>
<tr>
<td>4.0</td>
<td>0.33D</td>
<td>0.40D</td>
</tr>
<tr>
<td>5.0</td>
<td>0.35D</td>
<td>0.50D</td>
</tr>
</tbody>
</table>
The normal pressure \( p_{ni} \) acting on the wall is given by

\[
p_{ni} = k \ p_v
\]  \hspace{1cm} (3)

Hence from (2) and (3),

\[
p_{ni} = \frac{\gamma r_c}{\mu} \left[ 1 - e^{-(\mu k y/r_c)} \right] + p_{no} e^{-(\mu k y/r_c)}
\]  \hspace{1cm} (4)

where \( p_{no} = k \ p_{vo} \)  \hspace{1cm} (5)
Cylinder Pressures – Flow Case

In accordance with AS3774-1996, the flow pressure in the cylinder is obtained by multiplying the Janssen pressure by the factor $c_{nf}$.

$$p_{nf} = c_{nf} p_{ni}$$  \hspace{1cm} (12)

where $c_{nf}$ is the larger of

$$c_{nf} = [ 7.6 \left( \frac{h_d}{d_c} \right)^{n} - 6.4 ] \ c_c$$  \hspace{1cm} (13)

and $c_{nf} = 1.2 \ c_c$  \hspace{1cm} (14)

$h_c + h_d = \text{total effective height of stored solid}$
$c_c = 1 \text{ for axi-symmetric flow}$
$c_c = 1.2 \text{ for planar flow}$
$n = 0.06$
$d_c = D$
$p_{ni} = \text{Janssen pressure given by equation (4)}$
Hopper Pressures

\[
\frac{dp_{vh}}{dz_h} + \frac{j p_{vh}}{(h_h - z_h)} = \gamma
\]
Hopper Pressures

\[ k_h = \frac{p_{nh}}{p_{vh}} \]  \hspace{1cm} (15)

\[ \frac{dP_{vh}}{dz_h} + \frac{j P_{vh}}{(h_h - z_h)} = \gamma \]  \hspace{1cm} (16)

Solution of this equation leads to:

\[ P_{vh} = \gamma \left[ \frac{h_h - z_h}{j - 1} \right] + \left( \frac{\gamma}{j - 1} \right) \left( \frac{h_h - z_h}{h_h} \right)^{j} \] \hspace{1cm} (17)

The normal pressure on the hopper wall is

\[ P_{nf} = k_h P_{vf} \] \hspace{1cm} (18)

where

\[ j = (m+1) \left\{ k_h \left( 1 + \frac{\tan \phi_W}{\tan \alpha} \right) - 1 \right\} \]  \hspace{1cm} (19)
Hopper Pressures - Initial Filling Case

For the initial filling case, the minimum value of $k_h$ (that is, $k_{hi}$), is used. For this case, $j = 0$ and the vertical pressure $p_{vhi}$ is hydrostatic.

$$p_{vhi} = p_s + \gamma zh$$

and the normal pressure $p_{nhi}$ is

$$p_{nhi} = k_{hi} p_{vhi} = k_{hi} (p_s + \gamma zh)$$

with $j = 0$,

$$k_{hi} = \frac{\tan \alpha}{\tan \phi_w + \tan \alpha}$$
Stress Field in Hopper - Flow Condition

(a) Flow Channel in Hopper

(b) Yield Loci
Hopper Pressures - Flow Case

Equations (17) and (18) apply. That is

\[ p_{vhf} = \gamma \left[ \frac{h_h - z_h}{j - 1} \right] + \left[ (p_s - \gamma \frac{h_h}{j - 1}) \left( \frac{h_h - z_h}{h_h} \right)^j \right] \] \hspace{1cm} (23)

\[ p_{nhf} = k_{hf} p_{vhf} \] \hspace{1cm} (24)

and

\[ j = (m+1) \left\{ k_{hf} \left( 1 + \frac{\tan \phi_w}{\tan \alpha} \right) - 1 \right\} \] \hspace{1cm} (25)
Pressure Ratio $k_{hf}$ – Flow Case - AS3774-1996

\[ k_{hf} = \frac{1 + \sin \delta \cos 2\eta}{1 - \sin \delta \cos 2(\alpha + \eta)} \]  \hspace{1cm} (32)

\[ \eta = 0.5 \left[ \phi_W + \sin^{-1} \left( \frac{\sin \phi_W}{\sin \delta} \right) \right] \]  \hspace{1cm} (33)

\[ j = c_h [k_{hf} (\tan \phi_W \cot \alpha + 1) - 1] \]

\[ c_h = \begin{cases} 2 & \text{for a conical or pyramidal hopper} \\ 1 & \text{for a wedge or slot hopper} \end{cases} \]
Stockpile Surcharge Pressure

(a) Case 1 - Uniformly Consolidated Stockpile - Highly Incompressible Bulk Solid

\[ p_s = \gamma h_s \]

That is the effective head is equal to the actual head. The Rankine pressure or head is less conservative

\[ p_s = \gamma h_s \cos \theta_R \]

where \( \theta_R = \text{Angle of repose} \)
Stockpile Surcharge Pressure

Case 2 – Pre-Formed Rathole or Flow Channel

The effective head may be estimated using the Janssen equation. In this case the cylinder diameter is the actual rathole diameter $D_f$, and the wall friction angle is estimated on the assumption that the maximum shear stress occurs during flow. On this basis, $\phi$ is given by

$$\phi = \tan^{-1}(\sin \delta)$$

where $\delta$ = effective angle of internal friction

In many cases the $H/R$ ratio of the ratholes is such that the asymptotic value of the Janssen pressure may be applied. That is,

$$p_s = \frac{\gamma D_{e}}{4 \tan \phi k_j} \quad D_e = \text{Effective Rathole Diameter}$$