Fatigue of Welded Connections

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Fatigue

Fatigue is a process of accumulative damage produced by the fluctuation of stress and strains even when both stress and strains are below the static resistance level of the material. The damage is accumulating cycle by cycle and after a certain number of repetitions a failure will occur.

- The fatigue process is normally divided into three phases:
- Phase 1: crack initiation;
- Phase 2: crack growth;
- Phase 3: final fracture.

The following conditions and parameters are important on the quantification of the fatigue life:

- External cyclic loading;
- Loading mode with reference to the actual structural item;
- Time history of the external forces;
- Geometry of the item:
  - Global geometry of the item,
  - Local geometry at potential crack locus (driven by the Kt concentration factor);
- Material characteristics;
- Residual stresses;
- Production quality in general;
- Surface finish in particular;
- Environmental condition during service.
Fatigue Resistance of Welded Connections

The significant decrease on the fatigue resistance of the welded joint can be explained mainly by three factors:

- Severe notch effect due to the attachment and the weld filler metal;
- Presence of non-metallic intrusions or micro-flaws along the fusion line;
- Presence of large tensile residual stresses.
The maximum stresses in the distributions can be close to the yield stress of the material. In the tension area the effective mean stress will increase and the associated fatigue life decrease. As a result, the fatigue life of welded joints has limited sensitivity to the applied mean stress caused by the external loading.

Figure 2.9. S-N curve based on constant amplitude fatigue tests
The importance of the accurate evaluation of the stress can be seen keeping in mind that the number of cycles to failure is a function of the stress range level raised to the power of at least 3 for welded joint, therefore if the stress is underestimated, say, by 20%, the fatigue life will be close to 100% overestimated.

1. Nominal Stress. 2. Geometrical Stress. 3. Notch Stress.

At section A-A at the toe of the weld it is possible to define the geometrical stress (or hot spot stress, as refereed in the hot spot approach); \( \sigma_b \)

At section B-B away from the weld toe, the stress at the main plate corresponds to the nominal stress; \( S, \sigma_n \)

Finally at section A-A, it is also possible to find the notch stress \( (S, \sigma_{nw}) \)
Fatigue Assessment Approaches

Methodologies used to assess the fatigue life of welded components

- Nominal stress (Global Approach);
- Structural (Geometrical) or hot-spot stress (Local Approach);
- Notch stress (Local Approach);
- Notch strain (Local Approach);
- Crack propagation (Local Approach);
- Notch intensity (Local Approach).
Nominal Stress approach

The nominal stress approach consists basically in comparing the nominal stress range amplitude at the critical section of the component being evaluated with the S-N curve of the endurable nominal stress amplitude. The stress range amplitude is defined by:

$$\Delta \sigma_n = \sigma_{\text{max}} - \sigma_{\text{min}}$$

$$\sigma_{\text{min}} / \sigma_{\text{max}}$$

The fatigues curves are based on representative experimental investigations and include the following effects:

- Structural stress Concentration due to the detail shown;
- Local stress concentration due to the weld geometry;
- Weld imperfections consistent with normal fabrication standards;
- Stress Direction;
- Welding Residual Stresses;
- Metallurgical Conditions;
- Welding Process;
- Inspection procedure (NDT, non destructive Test), if specified;
- Post weld treatment, if specified.

The S-N curves together with detail classes of basic joints used by the nominal stress method which can be found in several standards and design guidelines and are mainly based on the statistical evaluation of relevant fatigue tests carried out in the 1970s.
Fatigue S-N Nominal Stress Curve

Figure 6: a) Design S-N curves for different type of Joints.  
b) Classification of Joints in FAT classes as per Eurocode, [2]

Each fatigue strength curve is identified by the fatigue class number FAT, which corresponds with the fatigue resistance of the detail at 2 million cycles. The slope of the curves based on normal stresses is $m=3.0$, the constant amplitude limit is $5 \times 10^6$ cycles ($m=5.0$ and the constant amplitude limit become $10^8$ for curves based on shear stress).

The S–N curve depends on the material, notch or detail class and weld quality class. This differs from the dependency on material, geometry and surface parameters in the case of non-welded members.
Nominal Stress Approach Summary

The fatigue curves are based on representative experimental investigations and include the following effects:

- Structural stress Concentration due to the detail shown;
- Local stress concentration due to the weld geometry;
- Weld imperfections consistent with normal fabrication standards;
- Stress Direction;
- Welding Residual Stresses;
- Metallurgical Conditions;
- Welding Process;
- Inspection procedure (NDT, non-destructive Test), if specified;
- Post weld treatment, if specified.

This approach is the most common method and the majority of the design codes include it. The nominal stress approach gives satisfactory results with a minimum calculation effort.

The following condition needs to be fulfilled to the successful application of the approach:

- The nominal stress can be well defined, not affected by macro geometric effects.
- The structural discontinuity is comparable to one of the classified details included in the codes.
- The detail is free of significant imperfections that reduce the fatigue strength of the detail.
- When the direction of the stress is parallel to the weld seam.
- In cases where the crack is expected to start at the root of the weld.
Hot Spot Stress Approach

The structural stress or strain used to assess the fatigue life of one component is called structural or geometrical stress.

It may be measured by strain gauges when the assessment is based on the strains and calculated by engineering formulas or finite element analysis when stresses are used as basis to do the evaluation.

The fatigue life in this approach is predicted comparing the intensity of the stress with the endurance limit given by a geometrical S-N curve.

The hot spot stress approach is used on applications where the fluctuation acts predominantly perpendicular to the weld toe (or the end of a discontinuous longitudinal weld), with the potential crack initiating at the weld toe or end.

The term hot spot refer to the critical point in a structure where a fatigue crack is expected to occur due to a discontinuity and/or notch.

The named ‘hot spot’ is due to the local temperature rise produced by cyclic plastic deformation prior to crack initiation.
Type of Hot Spots.

Two types of hot spot can be found, as shown in figure 7: Type “a” on which the weld is located on a plate surface and Type “b”, the weld is located on a plate edge.

Definition of structural stress.

The basic idea of the hot spot stress approach is to determine the structural stress at the toe of the weld, excluding the nonlinear component of the stress, which is referred in some codes (i.e. ASME) as the stress peak.

The reason to exclude the nonlinear portion of the stress is that at the design stage is not possible to know forehand the actual local weld toe geometry. The effect of the notch is implicitly included in the S-N curve.

Figure 7: Classification of Hot Spot Stresses, [3].
Parametric Determination of Structural or Hot Spot Stress.

a) Structural Stress Concentration Factor

A relation between the nominal stress and the structural or geometric stress can be established using an appropriated concentration factor that take into account the geometry of the connection.
Experimental Determination of the structural hot Spot

Experimental determination of the structural stress is based on strain measurement (using strain gauges). Once the strain is measured, it is possible to infer (Hook law) the stress on the surface of the component.

To avoid measuring any component of the notch stress (nonlinear portion of the stress), the measurement is undertaken some distance away from the weld toe.

Figure 9: Linear Extrapolation Procedure Type “a” Hot spot, [3].
The extrapolation procedure on hot spot type “b” is based on measurement of the strain at fixed distance: i.e.: 4, 8 and 12 mm. Based on these measurements the structural stress is found by quadratic extrapolation to the weld toe as follows:

$$\sigma_{\text{hr}} = 3 \cdot \sigma_{4 \text{ mm}} - 3 \cdot \sigma_{8 \text{ mm}} + \sigma_{12 \text{ mm}}$$

**Structural Stress Determination using Finite Elements.**

Finite element is an ideal tool for determining structural stress, due to its versatility to model different and complex structures. It also represents an excellent alternative to the experimental approach, since it is not required to have the physical model or the real component to be analysed.

Results must exclude the non-linear stress peak (as shown for Type ‘a’ hot spot in Fig. No 2.5), even in sections close to the weld toe. This will be the case with shell elements, since they automatically exclude the stress peak. If a single-layer solid element mesh is used, a linear distribution is obtained with 8-node elements or with 20-node elements after reduced
a) Determination Structural Stress.

In the case of the hot spot type “a”, it is possible to solve directly the linear stress distribution at the weld toe over the plate thickness and determine the membrane and bending components of the stress (see figure 8). Obviously this is the case when the stress distribution across the thickness is known.

Something that does not occur when shell elements are used. In this case extrapolation techniques similar to those applied to experimental stress analysis can be used.

As mentioned above, the use of a 3D finite element modelling of the connection including the weld seam, would allow determining the distribution of the stress at the weld toe by the linearization of the stress at that point.

However this approach requires having a high refine model to represent the weld profile in detail. For example the stress at the weld toe, due to the sharp notch it has, (which creates a singularity in the field stress) will increase toward the infinity as the element size become smaller. The foregoing is the reason why even using 3D models is common to use the extrapolation techniques to evaluate the hot spot stress.
b) Hot Spot Stress Extrapolation

Using coarse meshes, for hot spot type “a” when the mid points (for solid and shell elements) are used as extrapolations points, the following formula will be used to extrapolate the stress at the hot spot:

\[
\sigma_{hs} = 1.5 \cdot \sigma_{0.5t} - 0.5 \cdot \sigma_{1.5t}
\]

For hot spot type “b” when coarse mesh is used, the following relation is suggested:

\[
\sigma_{hs} = 1.5 \cdot \sigma_{5mm} - 0.5 \cdot \sigma_{1.5mm}
\]

Although strictly speaking each component of the surface stress has to be extrapolated to then resolve the principal stress at the hot spot, is a common practice to extrapolate the principal stress directly to the hot spot.

The stress determined using finite elements are dependant on the mesh density and the elements properties hence it is necessary to pay special attention on the selection of those two parameters. Figure 2.13 shows the recommendations give by the International Institute of Welding (IIW) on elements size and extrapolation points.
<table>
<thead>
<tr>
<th>Types of model and hot spot</th>
<th>Relatively Coarse Models</th>
<th>Relatively Fine Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element Size</td>
<td>Type a</td>
<td>Type b</td>
</tr>
<tr>
<td>Shell</td>
<td>( t_{xt}, \text{max} \times 0.5 \times L^* )</td>
<td>( 10 \text{mm} \times 10 \text{mm} )</td>
</tr>
<tr>
<td>Solid</td>
<td>( t_{xt}, \text{max} \times 0.5 \times L^* )</td>
<td>( 10 \text{mm} \times 10 \text{mm} )</td>
</tr>
<tr>
<td>Extrapolation Points</td>
<td>Shell</td>
<td>( 0.5 t / 1.5 t ) (mid-side points)</td>
</tr>
<tr>
<td>Solid</td>
<td>( 0.5 t / 1.5 t ) (surface centre)</td>
<td>( 5 \text{mm} / 15 \text{mm} ) (surface centre)</td>
</tr>
</tbody>
</table>

* \( L = \) attachment length (attachment + thickness + 2 weld leg lengths).
Structural Hot Spot S-N Curve.

The fatigue design curves are referred according to their FAT number, which correspond to the fatigue strength at $2 \times 10^6$ cycles for the hot spot stress range.

The general form of the S-N curve is shown in figure 2.15 and its equation is:

$$\Delta \sigma_{hs}^m \cdot N = C$$

Figure 10: S-N Curve for Hot Stress Approach, [3].
Modelling Guidelines Hot spot Method

Figure 11: Modelling recommendations for shell elements, [3].
Hot Spot Stress Summary

This approach is mainly used for connection where the weld toe is perpendicular to the principal stress and the crack is supposed to grow from the weld toe.

This approach is not recommended for cases where the cracks grow form the root of the weld or when the stress is parallel to the weld seam.

This approach is suited to be used under the following conditions:

a) There is not clear nominal stress due to geometric effects around the connection which cannot be isolated from the global effect of the loads.

b) The structural discontinuity is not comparable with anyone classified detail included in the nominal stress approach.

c) For the reasons mentioned in a) and b) finite element modelling is required to determine the field stress nearby the joint.

d) When testing of prototype is being used to evaluate the condition of the joint and strain gauges are used to estimate the stress at the joint.

e) Offset or angular misalignments exceed those specified at the nominal stress approach for the class of connection under study.
Crack propagation approach

Introduction to Linear Elastic Fracture Mechanic

The stress distribution at the front of the crack in a flat plate, as shown in figure 2.16 can be estimated using the Airy stress function with complex harmonic functions (see reference [1]). The stress field as function of the distance $r$ and an angle $\theta$:

![Stress distribution diagram](image)

Figure 12: Stress distribution at front crack tip, [11].
\[ \sigma_x = \frac{K_I}{\sqrt{2 \cdot \pi \cdot r}} \cdot \cos \frac{\theta}{2} \cdot (1 - \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2}) \]

\[ \sigma_y = \frac{K_I}{\sqrt{2 \cdot \pi \cdot r}} \cdot \cos \frac{\theta}{2} \cdot (1 + \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2}) \]

\[ \sigma_y = \frac{K_I}{\sqrt{2 \cdot \pi \cdot r}} \cdot \cos \frac{\theta}{2} \cdot \sin \frac{\theta}{2} \cdot \cos \frac{3\theta}{2} \]

\[ \sigma_z = \mu \cdot (\sigma_x + \sigma_y) \]
The stress intensity factor (SIF) $K_I$ relates the far field stress with the stress field ahead of the crack front. It is proportional to the applied stress and the square root of the crack size.

$$K_I = \sigma \cdot \sqrt{\pi \cdot a \cdot F(a)}$$

Where $F(a)$ is a geometry function that takes into account the geometrical deviation from the central through-thickness crack in an infinite plate. Depending on the loading modes the stress intensity factor can be classified as

![Diagram showing different loading modes: KI, KII, and KIII](image)
This approach, using fracture mechanics principles, for given a load histogram, allows to estimate the number of cycles require for an assumed initial crack to grow up to a critical size.

Basically this method, in its traditional form, neglects the crack initiation stage, which in many cases, taking into accounts the quantity of crack like defects present in a typical weld, results in a reasonable approximation. However, as it will be discuss later, in the case of high quality weld this assumption is not completely correct and it has been proved, although short, a nucleation stage is present at the majority of the cases.

The resistance of a material against cyclic crack propagation is characterized by the called Paris Law propagation which in its tradition form is represented by:

\[
\frac{da}{dN} = C \cdot (\Delta K_a)\text{m}
\]

\( C \) : Constant of the power law, material constant.
\( m \) : Exponent of the power law, material constant.
\( \Delta K \) : Range of stress intensity factor.
\( \Delta K_{th} \) : Threshold value of the stress intensity factor
\( a \) : Crack size, damage parameter.
\( N \) : Number of cycles.
The calculation of the stress intensity factor can be based on the nominal or geometrical stress at the location where the crack has to be determined. The stress should be separated into membrane and bending stress. The stress intensity factor $K$ is given by:

$$
K = \sqrt{\pi} \cdot a \cdot (\sigma_{\text{mem}} \cdot F(a)_{\text{mem}} \cdot M_{k_{\text{mem}}} + \sigma_{\text{ben}} \cdot F(a)_{\text{ben}} \cdot M_{k_{\text{ben}}})
$$

The effect of the weld notch or in other word the nonlinear peak stress is covered by an additional factor $M_k$

The stress gradient factor $M_k$ can be defined as the ratio between the $F(a)$ determined with notch, and $F(a)$ determined without a notch for a given crack:

$$
M_k(a) = \frac{F(a) \text{ with Notch}}{F(a) \text{ without notch}}
$$

This ratio can be determined using Finite Element Method (FEM), either directly by including the crack geometry in the FEM model or indirectly by an analysis of the body that does not have a crack.
7.1. Section of the wagon frame showing the hot spot location.
Table 7.1. Measured nominal stress range occurrences corresponding to a journey of 114 km. The journey included one loading/unloading cycle.

<table>
<thead>
<tr>
<th>i</th>
<th>$\Delta \sigma_i$ [MPa]</th>
<th>$n_i$ [cycles]</th>
<th>i</th>
<th>$\Delta \sigma_i$ [MPa]</th>
<th>$n_i$ [cycles]</th>
<th>i</th>
<th>$\Delta \sigma_i$ [MPa]</th>
<th>$n_i$ [cycles]</th>
<th>i</th>
<th>$\Delta \sigma_i$ [MPa]</th>
<th>$n_i$ [cycles]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73.5</td>
<td>0</td>
<td>9</td>
<td>55.1</td>
<td>4</td>
<td>17</td>
<td>36.7</td>
<td>6</td>
<td>25</td>
<td>18.3</td>
<td>353</td>
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<tr>
<td>2</td>
<td>71.2</td>
<td>0</td>
<td>10</td>
<td>52.8</td>
<td>3</td>
<td>18</td>
<td>34.4</td>
<td>11</td>
<td>26</td>
<td>16.0</td>
<td>270</td>
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<tr>
<td>3</td>
<td>68.9</td>
<td>0</td>
<td>11</td>
<td>50.5</td>
<td>4</td>
<td>19</td>
<td>32.1</td>
<td>19</td>
<td>27</td>
<td>13.7</td>
<td>333</td>
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<tr>
<td>4</td>
<td>66.6</td>
<td>1</td>
<td>12</td>
<td>48.2</td>
<td>8</td>
<td>20</td>
<td>29.8</td>
<td>43</td>
<td>28</td>
<td>11.4</td>
<td>813</td>
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<td>5</td>
<td>64.3</td>
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<td>13</td>
<td>45.9</td>
<td>10</td>
<td>21</td>
<td>27.5</td>
<td>116</td>
<td>29</td>
<td>9.1</td>
<td>1055</td>
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<td>6</td>
<td>62.0</td>
<td>5</td>
<td>14</td>
<td>43.6</td>
<td>5</td>
<td>22</td>
<td>25.2</td>
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<td>6.8</td>
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<td>41.3</td>
<td>9</td>
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<td>8</td>
<td>57.4</td>
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<td>24</td>
<td>20.6</td>
<td>332</td>
<td>32</td>
<td>2.2</td>
<td>76578</td>
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</table>
7.2. Detail with double gussets used to calculate the men stress concentration.

7.3. Hot spot region of the FE model. A relatively fine element mesh was chosen because of the use of linear 8-node solid elements. Due to the plane of symmetry, the total thickness of the web consisted of two element layers.
Table 6.1: Hot spot S-N curves for welded joints in steel plates up to 25 mm thick

<table>
<thead>
<tr>
<th>Joint</th>
<th>Description</th>
<th>Quality</th>
<th>FAT</th>
<th>$\Delta\sigma_{R,L}$</th>
<th>$\Delta\sigma_{th}$</th>
<th>$\Delta\sigma_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Butt joint</td>
<td>As-welded, proved free from significant flaws by NDE 1.</td>
<td>100</td>
<td>5%</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Cruciform or T-joint with full penetration welds</td>
<td>K-butt welds, no lamellar tearing.</td>
<td>100</td>
<td>5%</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Non-load carrying fillet welds</td>
<td>Transverse non-load carrying attachment, not thicker than the main plate, as welded.</td>
<td>100</td>
<td>5%</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A3.1: Bilinear hot spot S-N curves for plates up to 25 mm thick, $N_a=2\times10^7$ and $N_a=2\times10^9$ cycles.

<table>
<thead>
<tr>
<th>FAT</th>
<th>$m_1$</th>
<th>$C_1$</th>
<th>$\Delta\sigma_{R,L}$</th>
<th>$\Delta\sigma_{th}$</th>
<th>$m_2$</th>
<th>$C_2$</th>
<th>$\Delta\sigma_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>3</td>
<td>$2.000 \times 10^{12}$</td>
<td>73.7</td>
<td>46.4</td>
<td>5</td>
<td>$4.308 \times 10^{13}$</td>
<td>18.5</td>
</tr>
</tbody>
</table>

a) Stress concentration factor, $K_s$

The finite element model yielded the following results:

$\sigma_m(1.6 \text{ mm}) = 1.442 \text{ MPa}$

$\sigma_m(4.0 \text{ mm}) = 1.283 \text{ MPa}$

$\sigma_{hs} = 1.67 \cdot 1.442 - 0.67 \cdot 1.283 = 1.548 \text{ MPa}$

$\sigma_{hs} = 1.67 \sigma_{0.4t} - 0.67 \sigma_{1.0t}$

Use of relatively fine element meshing for analysing Type 'a' hot spots
Table 7.2. Damage calculation assuming no misalignment.
The bold line represents the knee point of the S-N curve.

<table>
<thead>
<tr>
<th>Level 4</th>
<th>$\Delta \sigma_{nom}$</th>
<th>$\Delta \sigma_{bg}$</th>
<th>$n_i$</th>
<th>$N_{fi} \times 10^6$</th>
<th>$n_i/N_{fi} \times 10^6$</th>
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<tr>
<td>4</td>
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<td>1</td>
<td>1.65</td>
<td>0.6050</td>
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<td>6</td>
<td>62.0</td>
<td>99.2</td>
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<td>2.4405</td>
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<td>110.75</td>
<td>2.9978</td>
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<td>25</td>
<td>18.3</td>
<td>29.3</td>
<td>353</td>
<td>200.18</td>
<td>1.7634</td>
</tr>
<tr>
<td>26</td>
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<td>25.6</td>
<td>270</td>
<td>361.81</td>
<td>0.6891</td>
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<tr>
<td>27</td>
<td>13.7</td>
<td>21.9</td>
<td>333</td>
<td>851.28</td>
<td>0.3912</td>
</tr>
<tr>
<td>28</td>
<td>11.4</td>
<td>18.2</td>
<td>813</td>
<td>2133.79</td>
<td>0.3810</td>
</tr>
<tr>
<td>29</td>
<td>9.1</td>
<td>14.6</td>
<td>1055</td>
<td>6500.00</td>
<td>0.1002</td>
</tr>
</tbody>
</table>

For assessments of structures subjected to variable amplitude loading, the fatigue damage is calculated according to the Miner's rule:

$$D = \sum \frac{n_i}{N_{fi}}$$

[A3.1]

where $D$ is fatigue damage;
$n_i$ is the number of cycles in the stress range bin $i$ (i.e., applied at stress range $\Delta \sigma_i$);
$N_{fi}$ is the fatigue life obtained from the appropriate S-N curve at $\Delta \sigma_i$. 

$\Delta n/N_{fi} = 41.4563 \times 10^6$
b) Results for a perfectly straight web

Table 7.2 shows the damage calculation when no account is taken of misalignment. The nominal stress ranges have been multiplied by $K_s=1.6$, giving structural hot spot stress ranges $\Delta \sigma_{ls}$. The three lowest stress levels have been omitted in this case.

The total damage $D=\Sigma n_i/N=41.46 \times 10^{-6}$. Thus, the predicted life in this case becomes:

$$N = \frac{114 \text{ km}}{41.46 \cdot 10^{-6}} = 2750000 \text{ km}$$
Figure 4.3. Example of stress time series

Figure 4.4. Definition of stress cycles based on rain-flow method, Ref [1]
Figure 4.5. Example of cycle counting for a stress time series

Counted cycles: 2-3-2', 5-6-5', 7-8-7' and 1-4-9
COMBINATION OF NORMAL AND SHEAR STRESS

If normal and shear stress occur simultaneously, their combined effect shall be considered. Three cases may be distinguished:

a) If the equivalent nominal shear stress range is less than 15% of the equivalent normal stress range or if the damage sum due to shear stress range is lower than 10% of that due to normal stress range, the effect of shear stress may be neglected.

b) If the normal and shear stress vary simultaneously in phase, or if the plane of maximum principal stress is not changed significantly, the maximum principal stress range may be used.

c) If normal and shear stress vary independently out of phase, in damage calculation the damage sums shall be calculated separately and finally added. A Miner damage sum of $\sum D_i = 0.5$ or the usage of $1/2$ of the calculated life cycles is recommended.

Fracture mechanics crack propagation calculations should be based on maximum principal stress range.
**Insensitive Mesh Structural Approach / Master S-N Approach**

The two main attributes that characterised this method is the insensitivity and the use of a single master S-N to curve determinate the fatigue life.

The designation of insensitive is due to the fact that the determination of the structural stress, providing that the geometry of the component under study is reasonably well captured within a finite element model, can be determine on an insensitive way respect to the type and size of elements.

The concept of collapsing all the parallel curves in one single master curve has a rational based.

Basically when the notch stress at the weld toe is used to characterise the connection, the difference between joint of geometry and load modes disappear, since they are included in the magnitude of the notch stress.

The difference with the notch stress approach is that Dong’s structural stress definition does not take into account the nonlinear component of the notch stress, which is highly variable depending on the geometry of the weld (angle and radius at the weld toe).
If the structural stress play a similar role to the far field stress, the total process can be seen as a transformation process where from a complex geometric and loading conditions to a simple fracture specimen in which the complex loading and geometry are capture in the form of membrane and bending stress (see figure 3.2).

Thus the existing stress insensitive factor, known for simple cases, can be used in the evaluation of cracks originated at the weld notch.

Figure 3.2: $E^2S^2$ Transformation process; complex geometry and load mode, reduced to a simpler fractomechanic problem, [5].
Two different methods are given to determine the structural stress depending on the type of elements used to represent the connection. In the case of solid models since the stress distribution can be determined through the thickness of the base plate, the static equivalent membrane and bending stress can be obtained through integration of the stresses over the thickness.

A second alternative of determining the structural stress is imposing the equilibrium condition over the nodal forces at the critical section.

Shell or plates elements, present the inconvenient that near the singularity the results they offered converge to the solution provided by the shell theory used into the formulation of the element. This characteristic is reflected on the fact that these elements exclude the nonlinear component of the stress. It is on these elements that the structural method presents the main advantages with respect to the hot spot approach.

Similar to the hot spot approach, the E2S2 method postulates that the stress at any position through the thickness of the plate under study can be express as an equivalent stress distribution composed of a membrane and bending components. Therefore the structural stress at for example the toe of a weld connection can be expressed as:

\[
\sigma_s = \sigma_m + \sigma_b
\]
Solid Model

\[ \sigma_m = \frac{1}{t} \int_0^t \sigma_x(y) \cdot dy \]

\[ \sigma_m \cdot \frac{t^2}{2} + \sigma_b \cdot \frac{t^2}{6} = \int_0^t \sigma_x(y) \cdot y \cdot dy + \delta \int \tau_{xy}(y) \cdot dy \]

Figure 3.4: Monotonic stress distribution at weld toe, [5].
\[ \sigma_m \cdot w \cdot t = \sum_{i=1}^{n} F_{xi} \]  \hspace{1cm} (3.8)

\[ \omega_b \cdot w \cdot \frac{l^2}{6} - \sum_{i=1}^{n} F_{xi} \cdot y_i \]  \hspace{1cm} (3.9)

The summation of the forces is carried out above the total number of nodes from \( i = 1 \) to \( i = n \) at section A-A. The term \( y_i \) is the local coordinate to the node \( i \) from the local reference coordinate system.
Once the local nodal forces are known, equivalents line forces \((f_x', f_z')\) and moments \((m_y')\) calculated in a work equivalent manner can be defined. Then the structural stress can be simply calculated as:

Coordinate rotations and solving simultaneous equations:

\[
\begin{bmatrix}
  F_1 \\
  F_2 \\
  F_3 \\
  \vdots \\
  F_n
\end{bmatrix} =
\begin{bmatrix}
  l_1 & l_1 & 0 & 0 \\
  3 & 6 & 0 & 0 \\
  l_2 & (l_1 + l_3) & l_2 & 0 \\
  6 & 3 & (l_2 + l_3) & 6 \\
  0 & 6 & 3 & 6 \\
  0 & 0 & \ldots & \ldots
\end{bmatrix}
\begin{bmatrix}
  f_1 \\
  f_2 \\
  f_3 \\
  \vdots \\
  f_n
\end{bmatrix}
\]

Structural Stress:

\[
\sigma_z = \sigma_m + \sigma_b = \frac{f_y'}{t} + \frac{6m_y'}{t^2}
\]

\[
(\tau_z - \tau_m + \tau_b) = \frac{f_x'}{t} + \frac{6m_y'}{t^2}
\]
Fig. 6: Structural stress calculations for a tubular T-joint investigated by Zerbst et al. [10]. (a) T-joint geometry and loading conditions; (b) Four FE models with different element sizes; (c) Comparison of the current structural stress results along weld toe at chord.

Fig. 7: Mesh-size insensitivity demonstration for a plate to I-beam box joint used in [1] (also see Fig. 1): (a) FE models with drastically different element sizes; (b) comparison of structural stress distributions along weld toe on attachment plate.
3.2 Two Stage Crack Grown Model and S-N Master Curve

The prediction of the fatigue life in the insensitive stress approach is carried out using a variant of the crack propagation approach developed to be used with the structural stress definition used by this method.

The main problem with the tradition crack propagation approach is that for all those cases where the initial growing crack stage is not negligible, as in the case of high quality welds, this theory failed to predict the total fatigue life.

A second issue is the accurate selection of the initial parameters, since the method provided an estimation of the fatigue life, estimating the time that take to an initial crack to grow to reach a critical size.

\[
\frac{da}{dN} = C \left[ f_1(\Delta K)_{a/t \leq 0.1} \times f_2(\Delta K)_{a/t > 0.1} \right]
\]

\[
\frac{da}{dN} = C \cdot (M_{kn})^n \cdot (\Delta K_n)^m
\]

\[
N = \int_{\alpha=0}^{\alpha=\infty} \frac{da}{C \cdot (M_{kn})^n \cdot (\Delta K_n)^m}
\]

Fig 9: Validations of the structural stress K estimation for T-fillet joint using Glinka’s weight function method [8]: (a) Remote bending, (b) remote tension
Master S-N Curve Approach Using Mesh-Insensitive Structural Stress Method

- Consistent characterization of stress concentration at welded joints using SS
- Additional fracture mechanics considerations:
  - Thickness effects
  - Remote loading mode effects

\[
N = \int_{a/t}^{t/s} \frac{td(a/t)}{C (M_{kn})(\Delta K)^m} = \frac{1}{C} \cdot \frac{1}{t^{1/2}} \cdot (\Delta \sigma_f)^m I(r)
\]

\[
\Delta \sigma_f = C \cdot t^{2m} \cdot I(r)^m \cdot N^{-m}
\]

\[
\Delta S_s = \frac{\Delta \sigma_s}{t^{2m} \cdot I(r)^m}
\]

Master S-N Curve ($\Delta S_s$-$N$)
Eq. (8) uniquely describes a family of an infinite number of structural stress based S-N curves \( \Delta \sigma_s - N \) as a function of thickness effects \( t \), and bending ratio effects \( r \). If Eq. (8) provides a good representation of the fatigue behavior of welded joints, an equivalent structural stress parameter can be defined by normalizing the structural stress range \( \Delta \sigma_s \) with the two variables expressed in terms of \( t \) and \( r \) on the right hand side of Eq. (8):

\[
\Delta S_s = \frac{\Delta \sigma_s}{\frac{2-m}{t^{2m}}} \cdot \frac{1}{(r^m)}
\]  

(9)
Effectiveness of The Equivalent Structural Stress Parameter

By regression analysis, the mean line of Fig. 13b can be represented in the form of:

$$\frac{\Delta \sigma_s}{t^{2-m} I(r)^m} = C N^{-\frac{1}{m}} = 16308 \times N^{-3.333}$$  \hspace{1cm} (10)
3.1 Fatigue Analysis Procedure

The methodology to implement the master curve approach as defined by Battle is as follows:

**Step 1**: Determine a load history based on the information in the Users Design Specification. The load history should include all significant operating loads and events that are applied to the component.

**Step 2**: For each location along the weld joint subject to a fatigue evaluation, compute the membrane and bending stress distribution for each point in the load using the structural stress method.
Step 3: Compute the equivalent structural stress based on the membrane and bending stress determined in Step 2 for each point in the load histogram using the structural, using the following equation:

\[ S_k = \frac{\sigma_k}{t^{(2-m)/2m}} \cdot I(r)^{1/m} \]

Where

\[ \sigma_k = \sigma_{k,m} + \sigma_{k,b} \]

\[ R_{b,k} = \frac{\sigma_{k,b}}{\sigma_{k,m} + \sigma_{k,b}} \]

\( m = 3.6 \)

\[ I(R_{b,k}) = 0.294 \cdot (R_{b,k})^2 + 0.846 \cdot (R_{b,k}) + 25.815 \]
Step 4: Compute the cyclic equivalent structural stress range $\Delta S_{\text{range},k}$, for the $k^{th}$ cycle based on the stress histogram and the equivalent structural stress determined in Step 3. Using the Rain flow cycle counting method, define the total number of cyclic stress ranges in the histogram as $M$.

Step 5: Determine the permissible number of cycles, $N_k$, based on the equivalent structural stress range parameter for the $k^{th}$ cycle computed in step 4 using the equation shown below.

$$\log N = B \cdot \log(f_c \cdot f_t \cdot \Delta S_{\text{range},k}) + A$$

Where

$f_c$ is a factor to take into account the effect of the fluid environment, loading, frequency, temperature and material variables such as grain size and chemical composition.
Step 6: Determine the fatigue damage for the $k^{th}$ cycle

$$D_{f,k} = \frac{1}{N_k}$$

Step 7: Repeat Steps 4 through 6 for all stress ranges, $M_i$ identified in the cycle counting process in step 3.

Step 8: Compute the accumulated fatigue damage using the following equation. The location along the weld joint is suitable for continued operation if this equation is satisfied.

$$\sum_{i=1}^{M} D_{f,k} \leq 1.0$$

Step 9: Repeat steps 3 through 8 for each point along the weld joint that is subject to a fatigue evaluation.
Figure 4.2: Tube joint SAE fatigue committee competition, [9].
<table>
<thead>
<tr>
<th>Mesh</th>
<th>0.5 x t</th>
<th>1 x t</th>
<th>2 x t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh size (flat surfaces)</td>
<td>3.9 mm</td>
<td>7.8 mm</td>
<td>13 mm</td>
</tr>
<tr>
<td>Mesh size (corners)</td>
<td>3.75 mm</td>
<td>7.8 mm</td>
<td>15.3 mm</td>
</tr>
<tr>
<td>Number of Nodes</td>
<td>15,500</td>
<td>7050</td>
<td>2,115</td>
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<tr>
<td>Number of Elements</td>
<td>15,540</td>
<td>7,026</td>
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</tbody>
</table>
Figure 4.5: Crack location during fatigue test, [9].
Figure 4.7: Stress distribution along 4 x 4 weld line, 1 x t model.
Figure 4.11: Stress distribution along 2 x 6 weld Line, 0.5 x t, 1 x t and 2 x t models.
Figure 4.9: Stress distribution along 2 x 6 weld line, 0.5 x t, 1 x t and 2 x t model.
Figure 4.8: Stress distribution along 2 x 6 weld line, 1 x t model.
Figure 4.12: Von Mises stress distribution MPa. 1 x t model.
References


